

1. For the data: 1,3,5,2,6,7,9. Please

(1) Compute the sample mean

(2) compute the sample standard deviation

(3) Get the median and discuss whether the mean or median will make larger difference if I add another value 100 in the dataset, justify or explain your answer.

2. Suppose there are 6 red balls and 4 white balls in a urn.

(1) Define event $A = \{\text{First draw is a white ball}\}$, what is $P(A)$, probability of A?

(2) Now assume we do picking balls one by one without replacement, that is after picking one ball, we keep it out and do the next picking in the rest. Define event $B = \{\text{Second draw is a white ball}\}$, what is $P(B)$, probability of B? (Hint: you need do conditional probability on the first picking)

(3) Are A and B independent? Justify your answer.

3. In a group, there are 16 female and 20 male in the first row, there are 10 female and 6 male in the second row, there are 24 female and 14 male in the third row.

(a) Construct the contingency table first.

(b) What is the proportion of female in the group, $P(\text{Female})$?

- (c) What is the proportion of individuals in the second row, $P(\text{Row } 2)$?
- (d) Given you randomly choose an individual, which is male, what is the probability that he is in the third row?
- (e) In the first two rows, what is the proportion of female individual?
- (f) Are gender and rank of row independent? You can do any comparison you want such as check whether $P(\text{female}) = P(\text{female} | \text{first row})$ or not.

4. Suppose one NBA player can get one point in each free shot with probability 0.8. Now he has five times of free shots and it is reasonable to assume that the result in each shot has no impact on other shots. We want to describe the total points he can get in these shots.

- (1) Define the random variable with checking assumptions.
- (2) What is the probability of his getting 3 points?
- (3) What is the probability of his getting at least 2 points?
- (4) If we define an unusual event if the chance of the event is below 5%. Is it an unusual event that he lost all his shots?

5. Suppose that X is uniform distributed on $[1, b]$ with $E(X) = 5$.

- (1) Find the upper bound b .
- (2) What is $\text{Var}(X)$
- (3) $P(X < 3)$
- (4) $P(X > 5)$

6. If X is $\text{Bin}(200, 0.5)$

- (1) What is the expectation and variance of X ?
- (2) Which approximation is better? Poisson or normal, justify your answer.
- (3) Using your approximation, find the approximate probability of $X=102$.
- (4) Using your approximation, find the approximate probability of X no less than 108.

7. For a Poisson distribution X , suppose I know the variance of X is 5.

- (1) By property of Poisson, what is mean of X .
- (2) Compute $P(X \geq 3)$
- (3) Does Poisson have the property of lack of memory? List at least one name of distribution with such property.

8. Suppose the growth rate of a population is normal distribution with mean 50 and standard deviation 20. That is $X \sim N(50, 20^2)$

- (1) Compute the probability of growth being larger than 70.
- (2) Compute the probability of growth being smaller than 40
- (3) What is the rate of growth such that it touches the top 10% of growth level?
- (4) Find the range such that the growth is in the middle of 80% of growth level.

9. Suppose a random variable X has mean 20 and variance 25.

- (1) Give your approximation bound for $P(X > 50)$

(2) Give your approximation bound for $P(|X-20|>10)$

(3) Now I collect 64 i.i.d (independent and identical) sample data from the population of X , what is mean and standard deviation of sample mean, $\bar{X}$?

(4) Approximate the probability of $P(\bar{X}<21)$, please also write the reason that you can do this approximation.

10. A lightbulb's lifetime is exponential distribution with variance 4 years².

(1) If we define X as the lifetime, find $P(X>3)$ (Hint: you need first figure out the λ)

(2) Given one lightbulb has already been lighting 2 years, what is the probability that it will continue to work at least 2 more years?